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A New Solution of the Solar Neutrino Flux

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ABSTRACT

We report a new solution to explain the observed deficit of the solar neutrino flux by Homestake, Kamiokande II and III, GALLEX and SAGE experiments. We use the matter mixing and the helicity oscillation in the twisting magnetic fields in the sun. Our model predicts the short (seasonal) and long (11 years) time variations of the solar neutrino flux. Three kinds of data observed by Homestake, Kamiokande, GALLEX and SAGE detectors are reproduced well if the mixing angle and the squared mass difference are in the small area around $\sin^2 2\theta \simeq 0.01$ and $\Delta m^2 \simeq 1.3 \times 10^{-8} \text{eV}^2$.

1. Introduction

Three kinds of data of the solar neutrinos flux are now available from Homestake[1], Kamiokande II and III[2], GALLEX[3] and SAGE[4] experiments. These data have shown the deficit of neutrino flux with respect to the prediction of the standard solar models[5],[6],[7]. The different deficit rates with respect to detectors show the neutrino energy dependence of the observed flux. If we take all of these data seriously, there needs new physics. The MSW (Mikheyev-Smirnov-Wolfenstein) matter mixing is the most natural scenario[8] of new physics.

Another characteristic feature is that the Homestake experiment seem to see the anti-correlation between the number of sunspots and the counting rates. This anti-correlation was not observed by the Kamiokande II and III experiments. Various authors[9] have examined this and concluded that the anti-correlation was not significant. Recently, we proposed a new interpretation of the time profiles of the Homestake and Kamiokande II data[10]. We take the complex time dependence of the Homestake data as a real phenomena and consider that the data show both the short (seasonal) and long (11 years) time variations. We interpreted the short time variation is originated from the effect of the twist of toroidal magnetic fields and the long time variation (11 years) is due to the change of their magnitudes. The Kamiokande II data are the averaged ones for about one year so that no short time variation shows up.

We constructed a simple model of the twisting toroidal magnetic fields following the simulation by Yoshimura[11] and made the numerical analysis. We successfully reproduced the Homestake data as well as the Kamiokande II data[10]. Since we are intended to see purely the effect of the twisting toroidal magnetic fields, we did not take into account of the matter mixing effect. Thus our predictions did not have the neutrino energy dependence. As a result, the GALLEX and SAGE data are not be explained.

In this paper, we consider the model which have both mechanisms; the matter mixing and the helicity oscillation in twisting toroidal magnetic fields. Our concern

is to find a new kind of solution which explain both the deficit of the neutrino flux (the time averaged profile and the neutrino energy dependence of the flux) and the short and long time variations of the flux. We made the numerical analysis and examined the allowed region in $(\Delta m^2, \sin^2 2\theta)$ plain. We found that if $\Delta m^2 \sim 10^{-8} \text{eV}$ and $\sin^2 2\theta \sim 0.01$, all data are reproduced well.

In Sec.2, we discuss the evolution equation and its general features. A model of the twisting toroidal magnetic fields are briefly explained in Sec.3 and the general features of oscillation of this scheme is discussed in Sec.4. Numerical analysis and the allowed region are given in Sec.5. Sec.6 is devoted to the discussions.

2. The evolution equation

Our model consists of two Majorana neutrinos, ν_e and ν_μ which have the transition magnetic moment μ . In the following, we use ν_e and ν_μ for left-handed neutrinos and $\bar{\nu}_e$ and $\bar{\nu}_\mu$ for their anti-neutrinos, respectively. The evolution equation for neutrinos which fly along z axis under magnetic fields is given by

$$i \frac{d\psi}{dz} = \begin{pmatrix} V_e & s_2 \delta & 0 & -\mu B_T e^{i\phi} \\ s_2 \delta & -V_\mu + 2c_2 \delta & \mu B_T e^{i\phi} & 0 \\ 0 & \mu B_T e^{-i\phi} & -V_e & s_2 \delta \\ -\mu B_T e^{-i\phi} & 0 & s_2 \delta & V_\mu + 2c_2 \delta \end{pmatrix} \psi, \quad (1)$$

where $\psi^T = (\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu)$, $s_2 = \sin 2\theta$ and $c_2 = \cos 2\theta$ with θ being a mixing angle, $\delta = \Delta m^2 / 4E$ with E being the energy of neutrino and Δm^2 being the squared mass difference. The magnetic fields relevant to the helicity oscillation is the transverse component which are parametrized by $B_T e^{i\phi} \equiv B_x + iB_y$. If the twist of toroidal magnetic fields exists, ϕ has the z dependence. The matter potentials for ν_e and ν_μ are denoted by $V_e = G_F(2n_e - n_n)/\sqrt{2}$ and $V_\mu = G_F n_n/\sqrt{2}$ where G_F is the Fermi constant, n_e and n_n are number densities of electrons and neutrons in the

sun, respectively. For them, we take[12]

$$\begin{aligned} V_e &= 0.195 R_\odot^{-1} \exp[10.82 \sqrt{1 - z/R_\odot}] \\ V_\mu &= 0.018 R_\odot^{-1} \exp[10.82 \sqrt{1 - z/R_\odot}]. \end{aligned} \quad (2)$$

This is the extension of the OVV (Okun, Voloshin, Vysotsky) model[13]. The model without the matter mixing ($\theta = 0$ case) was discussed by various authors[14],[15],[10] by considering various types of twisting magnetic fields. The application of this model to the solar neutrino problem with a realistic model of twisting magnetic fields was made in Ref.[10].

The importance of the phase ϕ can be understood by changing phases[14], $\psi^T \rightarrow \tilde{\psi}^T = (e^{i\phi/2}\nu_e, e^{i\phi/2}\nu_\mu, e^{-i\phi/2}\bar{\nu}_e, e^{-i\phi/2}\bar{\nu}_\mu)$. Then, apart from the overall phase, $\tilde{\psi}$ obeys the following equation

$$i \frac{d\tilde{\psi}}{dz} = \begin{pmatrix} V_e & s_2\delta & 0 & -\mu B_T \\ s_2\delta & -V_\mu + 2c_2\delta & -\mu B_T & 0 \\ 0 & \mu B_T & -V_e - \phi' & s_2\delta \\ -\mu B_T & 0 & s_2\delta & V_\mu + 2c_2\delta - \phi' \end{pmatrix} \tilde{\psi}, \quad (3)$$

where $\phi' = d\phi/dz$. The important role of the phase ϕ lies in the fact that the variation of it works as a potential. Due to this, various kinds of resonance oscillations will occur as discussed by Akhmedov, Petcov and Smirnov[16], depending on sizes of parameters, $c_2\delta$, $s_2\delta$, μB_T and ϕ' . There are essentially two resonance oscillations of ν_e . In the following, we assume $\Delta m^2 > 0$ and $\cos 2\theta > 0$ because we are interested in the transition of ν_e .

(i) The MSW matter (flavor) oscillation

If μB_T is small in comparison with $s_2\delta$, the MSW matter oscillation occurs. The resonance condition for $\nu_e \leftrightarrow \nu_\mu$ oscillation is

$$V_e + V_\mu = (\Delta m^2/2E) \cos 2\theta. \quad (4)$$

When the adiabatic condition at the resonance point

$$\left| \frac{1}{(V_e + V_\mu)} \frac{d(V_e + V_\mu)}{dz} \right| \ll \frac{\Delta m^2 \sin^2 2\theta}{2E \cos 2\theta} \quad (5)$$

is satisfied, ν_e converts fully to ν_μ . If this condition is not respected, the conversion is not full and the transition rate is estimated by the numerical computation or the Landau-Zener formula. The $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ oscillation does not occur because the condition $V_e + V_\mu = -(\Delta m^2/2E) \cos 2\theta$ is not met.

(ii) The OVV helicity oscillation in the twisting magnetic fields

If $s_2\delta \equiv (\Delta m^2/4E) \sin 2\theta$ is smaller than μB_T , the OVV oscillation in the twisting toroidal magnetic fields occurs. In this case, the oscillation between two different flavor neutrinos occurs because Majorana neutrinos can have only transition moments. The resonance point is determined by

$$\begin{aligned} V_e - V_\mu &= (\Delta m^2/2E) \cos 2\theta - \phi' & (\nu_e \leftrightarrow \bar{\nu}_\mu), \\ V_e - V_\mu &= -(\Delta m^2/2E) \cos 2\theta - \phi' & (\bar{\nu}_e \leftrightarrow \nu_\mu). \end{aligned} \quad (6)$$

We consider the survival probability P of ν_e after passing the distance Δz . The rate is expressed by[17]

$$P(\nu_e \rightarrow \nu_e; \Delta z) = 1 - B_f \sin^2 \frac{\sqrt{(V_e - V_\mu - 2c_2\delta + \phi')^2 + 4(\mu B_T)^2}}{2} \Delta z, \quad (7)$$

where B_f is the blocking factor defined by

$$B_f = \frac{4(\mu B_T)^2}{(V_e - V_\mu - 2c_2\delta + \phi')^2 + 4(\mu B_T)^2}. \quad (8)$$

The above formula is valid only when the variation of $V_e - V_\mu + \phi'$ is small in comparison with μB_T . In this formula, the matter potential acts as a blocking factor. At the resonance point, the blocking factor disappears ($B_f = 1$) and the conversion occurs. The adiabatic condition

$$\left| \frac{d(V_e - V_\mu + \phi')}{dz} \right| \ll 4(\mu B_T)^2 \quad (9)$$

at the resonance point.

Since the energies of neutrinos spread from about 0.33MeV (pp neutrino) to about 10MeV (^8B neutrino), $\delta \equiv \Delta m^2/4E$ varies about factor 30 so that neutrinos may receive different types of effects. We look for the situation where the pp neutrinos receive mainly the MSW matter oscillation effect, while the ^8B neutrinos receive the OVV helicity oscillation effect in the twisting toroidal magnetic fields. In this situation, the time variations occur for ^8B neutrinos, but not for pp neutrinos. This possibility is realized when δ and μB_T are the same size.

3. A model of twisting toroidal magnetic fields

Here we give a brief summary of the model of twisting toroidal magnetic fields which is proposed in our previous papers[10]. As in Fig.2 in Ref.10, We assume two tori in the convective area of the sun; one in the northern hemisphere and the other in the southern hemisphere parallel to the equator. We assume that toroidal magnetic fields locate in the torus and toroidal magnetic fields twist along it. This twist is parametrized by a parameter X , the distance along the torus to wind once. As in Fig.5 in Ref.10, we parametrize the configuration of the torus in the sun's cross section. The torus is parametrized by the latitude of its center Δ , its radius a , the distance between the center of torus and the center of the sun b .

The latitude of the neutrino path is parametrized by λ which is between -7.25° (the southern hemisphere) and 7.25° (the northern hemisphere). Since λ is small, neutrinos pass through almost around the edge of the toroidal magnetic fields so that we assume that the strength of magnetic fields is constant along the neutrino path. We specify the position of the entrance z_0 of a neutrino to magnetic fields and the departure z_1 from it. They are given by

$$z_{0,1} \equiv b \cos(\Delta - \lambda) \pm \sqrt{a^2 - b^2 \sin^2(\Delta - \lambda)}. \quad (10)$$

The twist of toroidal magnetic fields is generated by the differential rotation of the sun and the global convection of the plasma fluid in the convective area. By

solving the dynamo equation, Yoshimura[11] showed that the development of the twist of the toroidal magnetic fields is the origin of the cyclic oscillations of polarity reversals in every 11yr. The direction of the twist depends on whether magnetic fields lie in the northern or the southern hemisphere. The global structure of the twist is determined by the Coliori's force which acts on the global convection. By comparing the simulation by Yoshimura with our simple model, we found that the variation of the phase ϕ' is given by[10]

$$\phi'(z) = -\text{sign}(\lambda) \frac{2\pi/X}{1 + (2\pi/X)^2 [b \cos(\Delta - \lambda) - z]^2}. \quad (11)$$

The characteristic feature is that $\phi' < 0$ if neutrinos pass through the northern hemisphere and $\phi' > 0$ if they do the southern hemisphere. This sign difference will give the important affect on the deficit rate of the neutrino flux[10]. Various parameters defining the toroidal magnetic fields are fixed by comparing our model with the simulation by Yoshimura, $\Delta = 15^\circ$, $a = 0.1694R_\odot$ and $b = 0.8813R_\odot$.

4. General features of oscillation

We see some qualitative features of the neutrino survival rate. In order to make some concrete arguments, we take $\sin 2\theta = 0.1$, $(\mu B_T) \sim 4 \times 10^{-10} \mu_B \text{kG} \sim 8/R_\odot$ and $X = \pi R_\odot$ (about two turns) so that $\phi' \sim \pm 2/R_\odot$. Also, we restrict λ in two cases $\lambda = 7^\circ$ (northern hemisphere, around September) and $\lambda = -7^\circ$ (southern hemisphere, around April). With these values, we obtain $z_0 = 0.758R_\odot$ and $z_1 = 0.991R_\odot$.

The survival rate is given as a function of y which is defined as

$$y \equiv \frac{(E/1\text{MeV})}{(\Delta m^2/1\text{eV}^2)} = \frac{3.5 \times 10^9}{R_\odot} \frac{E}{\Delta m^2/(1\text{eV}^2)}. \quad (12)$$

The evolution equation is solved from the center to the surface of the sun with the initial condition $\psi = (1, 0, 0, 0)$ at the center of the sun where ν_e is created. In

Figure 1, we show our result about the y dependence of the survival probability $P(\nu_e \rightarrow \nu_e)$ observed on earth. The solid (dotted) line represents the survival probability of ν_e when it passes through the northern (southern) hemisphere. The seasonal difference arises for $y > 2 \times 10^7$ where the effect of the twisting magnetic fields becomes dominant. In Figure 2, we show to which species ν_e transformed. Fig.2b(c) corresponds to the case where neutrinos pass through northern (southern) hemisphere. ν_e transforms mainly to ν_μ up to $y \sim 3 \times 10^7$. When y is greater than this value, ν_e transforms to $\bar{\nu}_\mu$. The results in these figures may be understood qualitatively as follows:

(i) $y < 2 \times 10^5$

The resonance condition of the flavor oscillation in Eq.(4) is not satisfied and the resonance oscillation does not occur.

(ii) $2 \times 10^5 < y < 10^6$

In this region, the resonance point is inside the sun and the adiabatic condition is satisfied. Thus, ν_e transforms fully to ν_μ . The resonance condition of helicity oscillation may be satisfied, but the resonance point is not in magnetic fields which are located in the region $0.758R_\odot < z < 0.991R_\odot$. Thus, the helicity conversion does not occur.

(iii) $10^6 < y < 3 \times 10^7$

In this region, the non-adiabatic flavor transition occurs. As in Figs.1b and 1c, a part of ν_e transforms into ν_μ .

(iv) $3 \times 10^7 < y < 5 \times 10^8$

Since μB_T is assumed to be around $8/R_\odot$, it is larger than $(\Delta m^2/4E) \sin 2\theta$. Thus the helicity transition $\nu_e \rightarrow \bar{\nu}_\mu$ occurs. The resonance condition is expressed by $Y_E = 0.177 \exp(10.82\sqrt{1-z/R_\odot})$ and the adiabatic condition in Eq.(9) is expressed by

$$\left| \frac{Y_E}{\log |Y_E|} \right| \ll 1, \quad (13)$$

where $Y_E = 1.8 \times 10^9 / y - \phi' R_\odot$. In order for the adiabatic condition is satisfied, $|Y_E| \ll 1$ is required. This means that the resonance point should be very close to the surface and thus the adiabatic condition is not satisfied. In this non-adiabatic case, the estimation of the transition probability is rather complicated. We see from Fig.1a that in this y region the seasonal difference appears due to the effect of the twist ϕ' . The role of ϕ' is to shift the resonance point. When neutrinos pass through the northern (southern) hemisphere, ϕ' is negative (positive) and the resonance point z_r is smaller (larger) than the non-twisting case. For example, for $y \sim 10^8$, the resonance point determined from Eq.(7) is $z_r \sim 0.81R_\odot$ for the northern hemisphere and $z_r \sim 0.83R_\odot$ for the southern hemisphere. Since the transition is non-adiabatic, the transition develops gradually until neutrino reaches to the surface. It turns out that for $y \sim 10^8$ and $\phi' < 0$ (northern hemisphere), the survival rate becomes minimum before neutrino reaches to the surface so it oscillates back at the surface. On the other hand, for $\phi' > 0$ (southern hemisphere) the minimum value is achieved at the surface. This is the reason why the seasonal difference appears.

(v) $10^9 > y > 5 \times 10^8$

In this region, the pure helicity transition in the matter discussed in Ref.10 is realized. Mostly, ν_e transforms to $\bar{\nu}_\mu$. The general tendency is that the suppression of ν_e flux is strengthen when neutrinos pass through the northern hemisphere ($\phi' < 0$) and is weaken when neutrinos pass through the southern hemisphere ($\phi' > 0$). The transition rate depends sensitively on the strength of magnetic fields as shown in Ref.10.

Since the average energy of pp neutrinos is 0.33MeV and that of ^8B neutrinos is 10MeV, there is about 30 times difference. The experimental data by Homestake, Kamiokande II and III, SAGE and GALLEX suggest that pp neutrinos are less suppressed than ^7Be and ^8B neutrinos. From Fig.1a, there seem two possible solutions: One is the region $y \sim (10^6 \sim 10^7)$ where the MSW matter oscillation works. The pp neutrinos are less suppressed than ^7Be and ^8B neutrinos and no

time dependence of fluxes appears. In this solution, ν_e transforms to ν_μ . The other is the region $y \sim (10^8 \sim 10^9)$ where the pp neutrinos are less suppressed and have small time dependence. On the other hand, the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrinos are more suppressed and two kinds of time dependencies, long (11yr) and shot (seasonal) oscillation appear. This is the region where we are seeking.

5. A new solution

In order to solve the evolution equation, we have to give the size of the twist ϕ' and the time variation of the strength of magnetic fields. Although the twist should exist, the size of ϕ' is an unknown parameter. Here we take $X = \pi R_\odot$. We guess the time variation of magnetic fields from the variation of the sun spot number, $B_T \propto \sqrt{\text{sunspotnumber}}$. We also assumed the maximum strength $(\mu B_T)_{\text{max}} = 10.5 \times 10^{-10} \mu_B \text{kG}$. With these choices of values, we can estimate the neutrino flux at each year and each season. For the latitude of the neutrino path, we consider $\lambda = 7^\circ$ (around September) and $\lambda = -7^\circ$ (around April). According to the simulation by Yoshimura, magnetic fields on neutrino paths from August to October are similar to those of September and thus the calculation with $\lambda = 7^\circ$ will valid for the period from August to October. Similarly, the calculation with $\lambda = -7^\circ$ will be valid for the period from March to May.

We now seek a solution in the region $y \sim 10^8$ following the discussion given in the previous section. We considered three experiments, Homestake, Kamiokande II and III and GALLEX. For Kamiokande II and III, we used the formula

$$P(\text{KII}) = P({}^8\text{B}) + 0.11\bar{P}({}^8\text{B}), \quad (14)$$

where P and \bar{P} represent the probabilities of ν_e and $\bar{\nu}_\mu$, respectively. The symbol ${}^8\text{B}$ in the parenthesis show the survival rate of ${}^8\text{B}$ neutrinos which is evaluated with use of the average energy $E_\nu = 10\text{MeV}$. The contribution from the anti-neutrino

arises because Kamiokande II and III detectors are sensitive to it also. GALLEX and Homestake detectors can catch other than ^8B neutrinos. We used the formula

$$P(\text{GALLEX}) = \frac{70.8}{132}P(pp) + \frac{3.0}{132}P(pep) + \frac{34.3}{132}P(^7\text{Be}) + \frac{14.0}{132}P(^8\text{B}) + \frac{3.8}{132}P(^{13}\text{N}) + \frac{6.1}{132}P(^{15}\text{O}), \quad (15)$$

$$P(\text{Homestake}) = \frac{0.2}{7.9}P(pep) + \frac{1.1}{7.9}P(^7\text{Be}) + \frac{6.1}{7.9}P(^8\text{B}) + \frac{0.1}{7.9}P(^{13}\text{N}) + \frac{0.3}{7.9}P(^{15}\text{O}). \quad (16)$$

For pp , ^{13}N and ^{15}O neutrinos, the survival rates are estimated by using their average energies 0.33, 1.1 and 1.44MeV.

For Kamiokande II and III and GALLEX data, we demand that the time averages of our calculated rates agree with their averages within 1 standard deviation. We relaxed this constraint for the Homestake data because the data points changes with time so that the time average may not give a good measure for agreement. We put the importance on the time variation more than the average value for this case. We require that the time average of our estimates agree with the average experimental value within four standard deviation.

Allowed regions are shown in Figure 3. The area covered by dash-dotted lines, solid lines and dotted lined are the allowed regions by GALLEX data, Homestake data and Kamiokande II and III data, respectively. We found an unique shaded area which satisfies three experimental restrictions. The mixing angles and the squared mass in the allowed domain are $\sin^2 2\theta \sim 1 \times 10^{-2}$ and $\Delta m^2 \sim 1.3 \times 10^{-8}\text{eV}^2$.

We show the comparison between our predictions and the data in Figure 4. Fig.4a show the comparison with Homestake data, Fig.4b with Kamiokande II and III data and Fig.4c with GALLEX data. Our predictions for the Homestake data show the short (seasonal) and long (11yr) time variations and reproduce the data pretty well. We also predict these time variations for the Kamiokande II and III

data, but the comparison with respect to the short time variation will be premature at present because the Kamiokande II and III are given as the average for about one year. There appears no seasonal time variation if one year average data are taken. As for the GALLEX data, the main component is pp neutrinos. Thus we predict almost no time variation for GALLEX data as seen in Fig.4c. The agreements of our predictions and three kinds of data seem to be pretty good.

6. Discussions

We presented a new solution to explain the solar neutrino problems, the missing flux problem and the short and long time variations. We used a simple model of twisting magnetic fields in the sun and showed that this type of magnetic fields can give a new kind of solution for the solar neutrino problem. Our model predicts the short and long time variations which should appear in the Homestake and Kamiokande II and III data, and weak time dependence for GALLEX and SAGE data. Our predictions seems to reproduce these data pretty well. We also estimated to which type of neutrinos ν_e converts. From Figs.1b and 1c, the pp neutrino converts mainly to ν_μ , while the ${}^7\text{Be}$ and ${}^8\text{B}$ neutrinos do to $\bar{\nu}_\mu$. Thus, the situation in our model is more complicated than the MSW matter oscillation case. In order to clarify these situations, we have to wait the future experiments.

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FIGURE CAPTIONS

- 1) Energy dependence of the survival probability of ν_e . We estimated it for $\sin 2\theta = 0.1$, $(\mu B_T) \sim 4 \times 10^{-10} \mu_B \text{kG}$ and $X = \pi R_\odot$. The solid (dashed) line corresponds to the case where neutrinos pass through the northern (southern) hemisphere. The MSW matter oscillation mechanism dominates when $y \equiv (E/1\text{MeV})/(\Delta m^2/1\text{eV}^2)$ is smaller than 3×10^7 so that no seasonal difference arises. When $y > 3 \times 10^7$, seasonal differences appear due to the twist of toroidal magnetic fields
- 2) Energy dependence of neutrino components. We show where ν_e transforms when neutrinos pass through the northern hemisphere (a) and the southern hemisphere (b). For $y < 3 \times 10^7$, ν_e transforms mainly to ν_μ , while for $y > 3 \times 10^7$, ν_e does to $\bar{\nu}_\mu$.
- 3) Allowed domains by three experiments. Homestake data restrict the area to the domains surrounded by the solid lines; a large area in the upper-right corner and many small islands. Kamiokande II and III data restrict to the areas surrounded by the dotted lines; relatively large areas with $\Delta m^2 \sim 10^{-8} \text{eV}^2$ and $\sim (2 \sim 3)10^{-9} \text{eV}^2$ and several islands. GALLEX data restrict to the large area from $\Delta m^2 \sim 3 \times 10^{-7}$ to $4 \times 10^8 \text{eV}^2$ which is surrounded by the dash-dotted lines. The intersection of these areas is the small area with the shade.
- 4) The comparison of our predictions with three data. With the choice of some values of Δm^2 and $\sin 2\theta$ in the shaded area, we estimated the neutrino yields for three experiments. The comparison of our prediction with Homestake data is shown in (a), with Kamiokande II and III data in (b) and with GALLEX data in (c).

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